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Gray level transformation to increase the density of interferometric fringes

Mansour A. Ahmadshahi

A gray level transformation is presented to simulate the interferometric process. The transformation uses properties of sinusoidal functions to produce rapidly varying intensities from those with nearly zero gradients. The transformation when used in conjunction with optical techniques, such as holographic interferometry, has the effect of increasing the optical sensitivity and producing a large number of fringes where otherwise only a fraction of a fringe would be observed. This technique is ideal for holographic analysis of deformations in microscopic regions.

I. Introduction

Optical techniques such as holographic interferometry are utilized as extremely sensitive and accurate full field methods to determine displacements and strains.[1,2] Numerous signal processing techniques have been developed to retrieve phase information from interferometric fringe patterns.[3,4] As in the case of double exposure holography, once the fringes are formed and recorded on the holographic medium, be it emulsion or thermoplastic type, the recorded image is then routed to an image digitizer via a video camera where it is digitized and stored in a frame buffer for further analysis.

The deformations producing the fringe pattern together with the sensitivity of the optical setup are ordinarily large enough to generate many fringes in the field of analysis. One of the most difficult and seemingly unresolved aspects of accurate determination of deformations, however, occurs when the phase variation in the field of analysis is less than a complete cycle. This is because, in almost all circumstances, extraction of phase information from signals (be it analog or digital) containing low frequency components is subject to large errors. This problem was studied thoroughly in Ref. 5, and it was determined that its solution requires generating an auxiliary system of carrier fringes by optical means, and thus shifting the phase information to a higher frequency, that of the carrier fringes. The small perturbations of the phase of the carrier fringes can now be accurately detected by the use of FIR linear phase filters[6] and the appropriate techniques such as those outlined in Ref. 5.

In the present work we discuss a gray level transformation that can be applied to the recorded intensity variation that mimics the formation of fringes in optical processes. The transformation can be implemented on an image processor as the first stage of image acquisition, provided that the recorded holographic interferometric intensity has been optically filtered so that it does not contain a speckle pattern. If, however, the fringe pattern suffers from random intensity variation of speckle, the transformation can be applied in the second stage of image acquisition, after the fringe pattern has been numerically filtered.

It should be noted that, although the transformation is intended to be applied on interferometric images, it is not limited to them. It can be used to detect minute intensity changes, both qualitatively and quantitatively, since, as is shown, the transformed images are sinusoidal in form and numerous signal processing algorithms have been developed to process these types of image.

II. Formation of Holographic Interferometric Fringes

In a typical off-axis two-beam holographic moire setup[7] (see Fig. 1), the object is illuminated by two symmetric collimated beams, each making an angle α with the normal of the object plane. The scattered waves emanating from the object together with the collimated reference beam are then collected on the holographic plate, and a first exposure is taken. After

The author is with University of California, San Diego, Department of Applied Mechanics & Engineering Sciences, Center of Excellence for Advanced Materials, La Jolla, California 92093.
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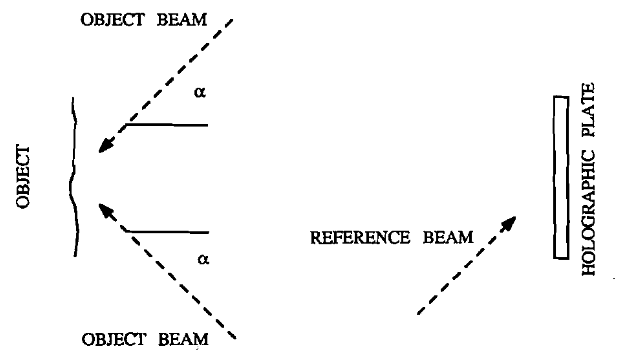

Fig. 1. Schematic representation of a typical off-axis two-beam holographic moire setup.

deformation of the object, a second exposure is taken and the resulting intensity variation can be written as

$$I(x,y) = B(x,y) + C(x,y)\cos[ku(x,y)], \tag{1}$$

where

$$k = \frac{4\pi \sin\alpha}{\lambda}, \tag{2}$$

and it is the sensitivity of the optical setup, λ being the wavelength of light. The spatially varying functions $B(x,y)$ and $C(x,y)$ determine the background and the contrast of the image, respectively.

Accordingly, an alternative form of Eq. (1) can be generated by stepping the phase of the reference beam by $\pi/2$, and thus producing an in-quadrature signal as

$$H(x,y) = B(x,y) + C(x,y)\sin[ku(x,y)]. \tag{3}$$

In fact, there are numerous numerical[5] and optical[8] techniques to generate signals of the forms in Eqs. (1) and (3). In Ref. 8, for example, the method involves obtaining three interferograms, using a CCD array, that have phase differences of $\pi/2$ and π, such that they facilitate the decoupling of the background function $B(x,y)$ and the amplitude term $C(x,y)$. It is therefore possible to normalize, say, Eq. (3), and obtain a digital image of the form

$$q(x,y) = A\{1 + \sin[ku(x,y)]\}, \tag{4}$$

where A is a constant and is one-half of the maximum gray level of the digitized image. If, for example, an 8-bit digitizer is used, the value of A would be 255/2.

III. Gray Level Transformation

There are numerous gray level transformations in the literature which are designed to enhance the gradients of the recorded intensities. One of the closely related algorithms is the well-known sawtooth gray scale transformation,[9] where equidistant gray levels are stretched linearly from 0 to 255 (assuming an 8-bit digitizer), enhancing the gradients throughout the image. Here, we use a sinusoidal gray scale transformation, which converts constant levels of intensities into minima of a sinusoidally varying intensity.

One of the intrinsic properties of the holographic interferometry process is that the changes of the path length of light are tremendously amplified as they become the argument of a sinusoidal function [see Eq. (4)]. In this equation, $u(x,y)$, a component of the in-plane deformation, is actually the change in the path length of light traveled from some arbitrary datum to the recording medium. It is constructive to interpret Eq. (4) as the transform of the deformation $u(x,y)$. In this way, a pair of transforms can be written as

$$M_c^{\omega,A}[\ ] = A\{1 + \cos\omega[\ ]\}, \tag{5}$$

$$M_s^{\omega,A}[\ ] = A\{1 + \sin\omega[\ ]\}. \tag{6}$$

By looking at interferometric fringes as the transforms of small path length changes of light, the above transformations can be simulated and operated on digitized intensities to mimic the process that takes place optically. It is such a transformation that produces rapidly varying functions from those with nearly zero gradients. A table of gray scale transformations can be made and loaded onto the lookup table of an image processor. In this way, a transformed image can be recorded whose intensity gradients are greatly amplified.

To illustrate the proposed algorithm, we consider the application of this gray scale transformation to the intensity distributions of two objects emanating lightwaves of completely different natures. Their digitized intensity variations have been transformed according to Eqs. (5) and (6). While the first example is only a demonstrative one, the second example deals with detecting extremely small deformations of a circular disk under diametrical compression.

Figure 2 shows the digitized image of a Ping-Pong ball being illuminated by the ceiling light. Figures 3 and 4 show the transformed versions of the same image when the input lookup table of the image processor was loaded according to Eqs. (5) and (6), respectively. The contours of constant intensity are clearly evident in these figures. Here, we have chosen the parameters $A = 127.5$ and $\omega = 2\pi\xi/255$, with ξ being equal to 8. This parameter determines the number of fringes that are produced in a region whose intensity changes from 0 to 255. For example, a line of 256 pixels, whose intensity changes linearly from 0 to 255, can be transformed to modulate sinusoidally with the maximum number of oscillations being 128. Since there are only 256 discrete gray levels, a sinusoidal transformation can only generate 128 distinct levels of gray. One must also consider that the number of fringes generated by this transformation does not exceed the maximum number of fringes that can be properly represented in a given region if aliasing is to be avoided. It is therefore recommended that low values of ξ be used at first and then gradually increased.

The second example describes the feasibility of the use of this transformation to increase the density of fringes obtained in any interferometry process. Suppose that in a given field of analysis the deformations are so small that the change in phase is only a fraction of a cycle and that the image has been, optically or numerically, normalized as

$$I(x,y) = A\{1 + \sin[ku(x,y)]\}, \tag{7}$$

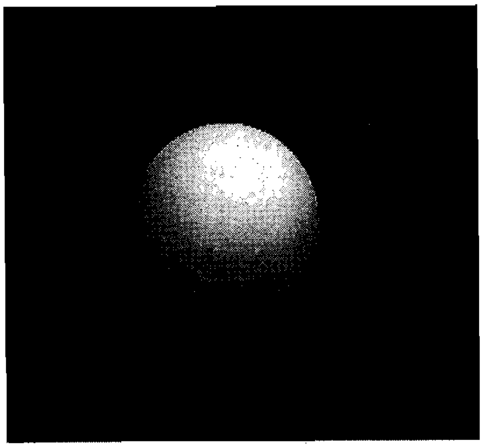

Fig. 2. Intensity variation of a Ping-Pong ball illuminated by white light.

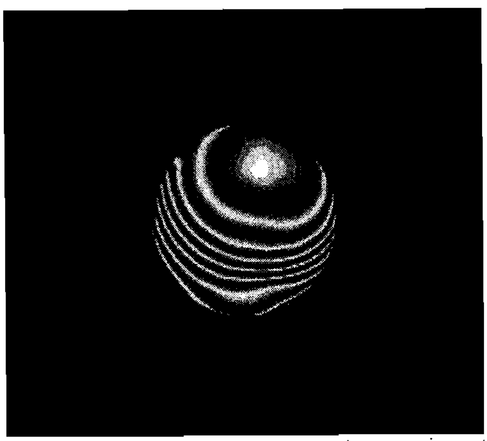

Fig. 3. Transformed intensity variation of Fig. 2. using Eq. (5) with $\xi = 8$.

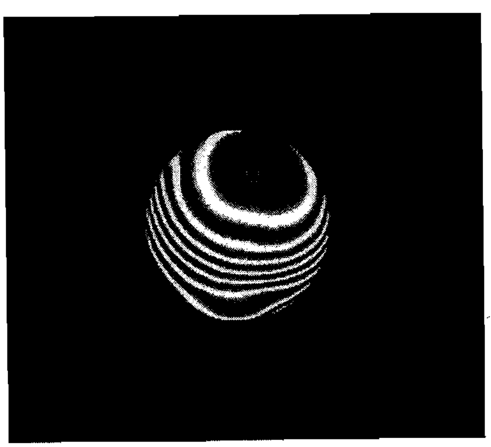

Fig. 4. Transformed intensity variation of Fig. 2. using Eq. (6) with $\xi = 8$.

where A is a constant and is the dc level of the digitized image, and k is also a constant and is the sensitivity of the optics. If the argument of the sine function is small, say, less than $\pi/6$, it can be written as

$$I(x,y) = A\{1 + [ku(x,y)]\}. \tag{8}$$

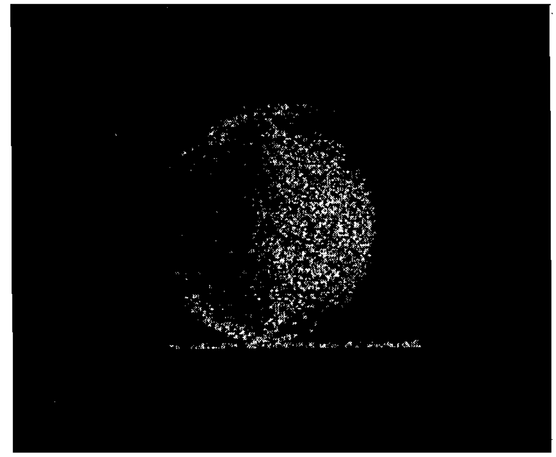

Fig. 5. TV holographic moire fringe pattern of a circular disk under a small diametrical compressive load.

The error introduced by replacing the sine function with its argument up to this range is <5%. Applying one of the transformation, say, Eq. (6) or Eq. (8), produces the following intensity distribution:

$$\begin{aligned} q(x,y) &= M_s^{\omega,A}[I(x,y)] \\ &= A\{1 + \sin[\omega I(x,y)]\} \\ &= A\{1 + \sin[\omega A + \omega A k u(x,y)]\}. \end{aligned} \tag{9}$$

Except for an inconsequential phase change, ωA, Eq. (9) resembles the intensity variation of Eq. (7), but the sensitivity factor has been amplified by the factor ωA.

Figure 5 shows the holographic moire fringe pattern of a circular disk under a small diametrical load obtained by the TV holography technique. The specimen is made out of stainless steel with $E = 30 \times 10^6$ psi and $\upsilon = 0.3$. The diameter of the disk is 3.8 cm (1.5 in.) and its thickness is 0.64 cm (0.25 in.) It is loaded vertically using a computer-controlled Instron machine. Under load feedback control, a load of 20 ± 1 lb was applied. The optical setup was such that the corresponding double illumination angles α, shown in Fig. 1, were equal to 45°. An argon laser of $\lambda = 514$ nm was used as the light source. Because of symmetry, only the first quadrant of the disk was analyzed. As can be seen in Fig. 5, the image is greatly corrupted by the presence of speckle. This image was filtered by 2-D and 1-D FIR linear phase low pass filters to remove the random intensity changes of the speckle pattern. A composite image of the processed experimental results, together with the theoretical fringe pattern, is shown in Fig. 6. The contrast is assumed to be unity and the dc level A of Eq. (4) was calculated by computing the average intensity over the circular region before loading, and a value of 250 was obtained. These values were used when generating the theoretical segment of the image in Fig. 6. Figure 7 shows the transformed images of Fig. 6 when Eq. (5) is used with $A = 255$ and ω has values of $2\pi\xi/250$ with $\xi = 8$ and $\xi = 32$. It can be seen that by applying the transformation we have generated high density fringe patterns of an initial image

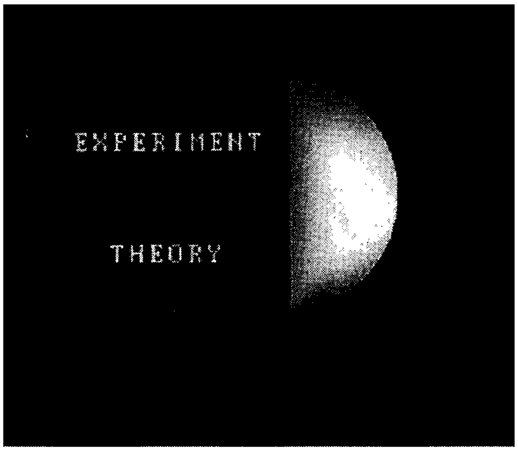

Fig. 6. Fractional fringe patterns: top represents the experimental fringe pattern after removal of speckle noise; bottom represents the theoretical solution.

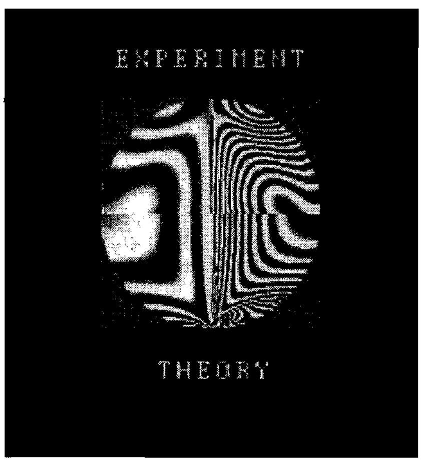

Fig 7. Artificial fringes generated by applying the transformation of Eq. (6) on the image shown in Fig. 6. The first and second quadrants show the experimental results for $\xi = 32$ and $\xi = 8$, respectively. The third and fourth quadrants show the theoretical results for $\xi = 8$ and $\xi = 32$, respectively.

(Fig. 6) whose phase variation is only a fraction of a cycle throughout the field.

IV. Conclusions

A gray level transformation has been developed that mimics the interferometry process. The property of this transformation is that it amplifies the gradients in the intensity distribution of an image. A distinct advantage of this transformation is that the transformed intensities are in the form of sinusoidal functions, which are readily understood, and whose digital processing is thoroughly formulated. Two examples have been given to demonstrate the applicability of the algorithm. The first example has been given with the intent of familiarizing the reader with the transformation. The second example has been given to obtain high density fringe patterns from the recorded interferogram of a circular disk under an extremely small compressive load using the TV holography technique. The results have been compared with the theoretical solution and good agreement has been shown to exist, verifying the validity and the applicability of the transformation.

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